SURFACE TENSION CORRECTION IN THE PRECESSION OF THE PERIHELION OF MERCURY

A Term Paper

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In Partial Fulfillment for the Requirement of Master's Degree of Science in Physics

By

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RECOMMENDATION

This is to certify that Mr. Raju Sharma Khatiwada has carried out the research work entitled "**SURFACE TENSION CORRECTION IN THE PRECESSION OF THE PERIHELION OF MERCURY**".

I recommend the term paper in the partial fulfillment for the requirement of Master's Degree of Science in Physics at GoldenGate Int'l College, Tribhuvan University, Kathmandu, Nepal.

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Last but not the least I am indebted to the entire legacy of physicists and scientists, whose giant shoulders I rest upon, assured to make advances of my own.

Raju Sharma Khatiwada

EVALUATION

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ABSTRACT

We study the implications of the surface tension in general relativity. The investigation is carried out by means of the motion of the planets. In this paper, we explore the effect of surface tension in the precession of the perihelion of mercury.

Keywords: Surface, Tension, General Relativity, Precession of Perihelion of Mercury

CONTENT

CHAPTER 1

INTRODUCTION

Chapter 1
Introduction

The Einstein's General Theory of Relativity (GR) was not an easy theory to test. Einstein himself laid out three ways to verify the GR observationally. These include **i).** The bending of light in gravitational field **ii).** The gravitational redshift and **iii).** The precession of the perihelion of the Mercury. The test of precession of perihelion of mercury was carried out by **Dyson and Eddington** [1] who conducted extensive observations of the solar limb during 1919 and 1920.

1.1 Perihelion shift of Mercury

Any object in a two-body system, would trace out an ellipse with the center of mass of the system at the focus of the ellipse. The point of closest approach of a planet with respect to the sun is called the perihelion. Due to the perturbing effect of other planets in the solar system, the major axis of the elliptical mercury orbit rotates around the sun. This is called the precession of the perihelion of mercury. Not just mercury but all planets revolving around the common center of mass with the sun should exhibit this precession. But since the perihelion shift from equation **1)** is inversely proportional to the square of the time period of the orbit, other planets in the solar system show very little perihelion shift because of their larger periods.

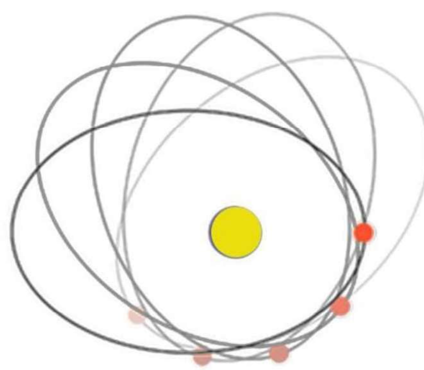

Fig 1: Graphical representation of Perihelion shift. As time passes, the perihelion of mercury precesses from the brighter red to the fainter red. The central yellow spot is the sun.

$$\sigma = \frac{24\pi^3 l^2}{T^2 c^2 (1 - e^2)} \tag{1}$$

1.2 Solar Limb Effect

At the solar limb, there is an anomaly known as limb shift which is the wavelength shift of the Fraunhofer lines relative to the center. **Halm (1907)** [2] first found the solar wavelength shift by observing the two iron lines near 6302 Å which increased from disk center to the limb by 12 mÅ. This limb effect was confirmed for 14 lines by **Fabry & Buisson (1910)** and for 470 other Fraunhofer lines by **Adam (1910) [3].** Such wavelengths shift to the red is interpreted as the redshift which is basically an effect of the expansion of the universe. But if there is a mechanism of energy loss of travelling photons, we can reinterpret the meaning of wavelength shift.

1.3 Energy loss mechanisms in the solar limb

The observations of the steady Ca^+ and Na absorption lines show that a very dilute gaseous mass may exist in all of the interstellar space **[4]**. **Adam (1948) [5]** believed that collisions of photons with neutral hydrogen atoms would cause a shift towards the red.

Hubble Law tells us that the recessional velocity of galaxies is proportional to the distance. **Zwicky (1929) [6]** however, proposed that the redshift versus distance relation isn't linear and attributed to gravitational fields, the energy loss of photons. To support Zwicky's claim, **Ten Bruggencate [7]** concluded that redshift of Globular clusters was related to the gravitational field of intervening medium.

Keeping these discoveries into account, there can be, in fact, a new mechanism of energy loss in solar. From our observation of the solar limb effect, we know that the solar limb isn't completely transparent to the non-scattered radiations. The scope of Raman and Compton Scattering is discussed further in chapter 2.

New mechanism of energy loss of non-scattered photons is proposed by **Ortiz (2020) [8]** based on the mechanism of surface tension. It extends Zwicky's idea by proposing the actual mechanism of energy loss under surface tension.

1.4 Surface Energy

We aim to define a new Energy loss mechanism compatible with the observations. In this paper, we are proposing a new effect which is the effect of surface tension under gravitational potential. This force is the analogue of the surface tension under van der Waals potential. Perhaps, it is misnomer to define it as a surface energy because it is the cumulative energy loss of travelling photons due to the gravitational effect on the particles of the medium.

CHAPTER 2
LITERATURE REVIEW

Chapter 2

Literature Review

2.1 Modified Line Element and the associated metric

From the work of **Carlos Ortiz (2020) [8]**, we derive and use the expression for surface energy to modify the Schwarzchild line element. In order to study the effects of energy loss due to surface tension in GR, we need to select the corresponding line element that includes such characteristics. In general relativity redshift con be interpreted in different ways, these leads to several important cases depending on the space-time geometry and the given symmetries of the analyzed configuration.

The gravitational redshift is interpreted as a time dilatation due to the gravitational force. The redshift effect is associated to the Schwarzschild solution by,

$$1+z=\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}} \tag{2}$$

In another context, the redshift relation may be expressed in terms of escape velocity $v^2 = 2GM/R$, leading to the Lorentz factor,

$$1+z= \boldsymbol{\gamma_e} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3}$$

This way we can relate the velocity to the temporal component of the metricg_u.

From **Carlos Ortiz (2020) [8]**, the surface tension modified Friedman equation is,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3} + \frac{8\pi G}{3}\rho_{0r}\left(\frac{a}{a_0}\right)^{-4} + \frac{8\pi G}{3}\rho_0\pi\left(\frac{a}{a_0}\right) \tag{4}$$

We can obtain the velocity using the relation of the distance and the scale factor $r = a(t)r\hat{}$, neglecting the radiation term,

$$\dot{r}^2 = v^2 = 2\frac{GM}{r} + \frac{8\pi^2 G}{3}\rho_0\frac{r^3}{r_0} = \tag{5}$$

Substituting this in equation 3) gives,

$$1+z=\frac{1}{\sqrt{1-\frac{2GM}{rc^2}-\frac{8\pi^2 G}{3}\rho_0\frac{r^3}{r_0c^2}}} \tag{6}$$

Now, we relate the redshift due to the metric with equation 6). In doing so, we have related the surface tension effect to the metric.

Thus, $$g_u = 1 - \frac{2GM}{rc^2} - \frac{8\pi^2 G}{3}\rho_0 \frac{r^3}{r_0 c^2} \tag{7}$$

This is how we find the appropriate metric for our case.

With this idea, we modify the Schwarzchild line element as

$$ds^2 = \left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right)c^2dt^2 - \left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{8}$$

The additional term can also be understood as the work done against the Surface force from the figure below.

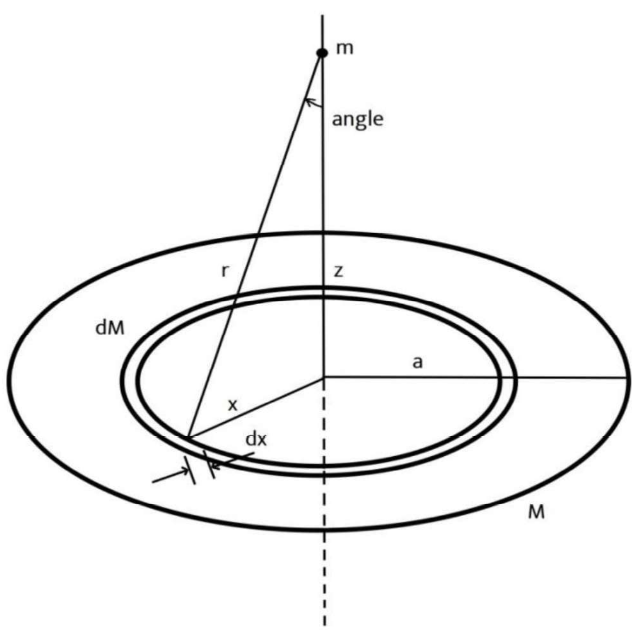

Fig 2: The mass distribution

2.2 Gravitational Drag to reconcile with GR

Any photon of energy $'h\nu'$ has an inertial and gravitational mass given by$'h\nu/c^2'$. When the photon passes through a gaseous medium of Mass M, it will not only be deflected but will transfer its momentum and energy. This energy loss corresponds to a frequency shift.

The observations of the steady Ca^+ and Na absorption lines show that a very dilute gaseous mass may exist in all of the interstellar space **[4]**. The cumulative mass of the particles of the medium is M.

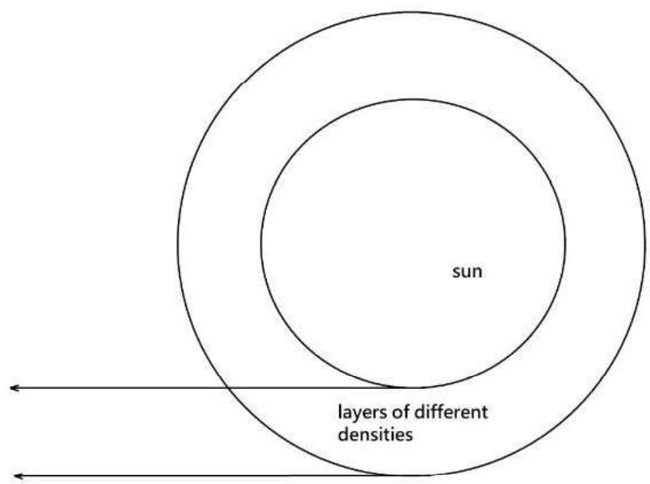

Fig: 3: Paths for light rays in regions of different densities

The differential shift due to the density of perturbing atoms can be an explanation of the solar limb effect.

The differential shift suggests that the solar corona is not completely transparent to non-scattered photons. For scattered photons, we can explain their redshift due to the Compton Scattering. This works only for single photon-electron interaction though the energy loss is very small compared to the equivalent energy of the corresponding observed redshift. For many photon-electron interactions, the multi-directional nature of scattering would make the universe extremely opaque. So, scatterings like Compton or Raman scattering can't account for the observed redshift of the photons. For the redshift of non-scattered photons, there can be a new mechanism of energy loss in the solar corona. We therefore are trying to develop a gravitational analogue of the Compton Effect by exploiting the Schwarzchild line element of GR.

CHAPTER 3

METHODOLOGY

Chapter 3

Methodology

3.1 Theory

To study this effect of energy loss due to surface tension in GR, we modify the Schwarzchild line element as,

$$ds^2 = \left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right) c^2 dt^2 - \left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{9}$$

where $R_s = \frac{2m_s G}{c^2}$ is the Schwarzschild radius of the Sun, m_s is the Sun's mass. Here the last term inside the bracket is the contribution due to the surface tension[**?**]. With the inclusion of this surface tension, the body (in this case mercury of mass μ revolves in its orbit with some effective observational velocity. Now we modify all other equations based on this contribution. The Lagrangian for a μ mass, becomes

$$L = \frac{\mu}{2}\left[\left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right) c^2\dot{t}^2 - \left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right)^{-1} \dot{r}^2 - r^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right)\right] \tag{10}$$

The corresponding momenta are obtained in the usual way

$$\Pi_t = \frac{\partial\mathcal{L}}{\partial\dot{t}} = \mu\left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right) c^2\dot{t}, \tag{11}$$

$$\Pi_r = \frac{\partial\mathcal{L}}{\partial\dot{r}} = -\mu\left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right)^{-1} \dot{r}, \tag{12}$$

$$\Pi_\theta = \frac{\partial\mathcal{L}}{\partial\dot{\theta}} = -\mu r^2\dot{\theta}, \tag{13}$$

$$\Pi_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = -\mu r^2\sin^2\theta\dot{\phi}, \tag{14}$$

so, we build the Hamiltonian as

$$H = \Pi_{q^i}\dot{q}^i - \mathcal{L} \tag{15}$$

for simplicity, we consider the plane $\theta = \frac{\pi}{2}$, then, $\dot{\theta} = \ddot{\theta} = 0$. Finally, the Hamiltonian function becomes

$$H = \frac{1}{2\mu}\left[\frac{1}{c^2}\left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right)^{-1} \Pi_t^2 - \left(1 - \frac{R_s}{r} - \frac{8G\pi^2\rho_0 r^3}{3r_0 c^2}\right)\Pi_r^2 - \frac{\Pi_\phi^2}{r^2}\right] \tag{16}$$

The dynamics of the systems is given by,

$$\dot{\mathrm{r}} = \frac{\partial\mathcal{H}}{\partial\dot{\Pi}_\mathrm{r}} = -\frac{\Pi_\mathrm{r}}{\mu}\left(1-\frac{\mathrm{R_s}}{\mathrm{r}}-\frac{8\mathrm{G}\pi^2\rho_0\mathrm{r}^3}{3\mathrm{r}_0\mathrm{c}^2}\right), \tag{17}$$

$$\dot{\Pi}_\mathrm{r} = -\frac{\partial\mathcal{H}}{\partial\dot{\mathrm{r}}} = \frac{1}{\mu}\left[\frac{\frac{\mathrm{R_s}}{\mathrm{r}^2}-\frac{4\mathrm{G}\pi^2\mathrm{r}^2}{\mathrm{c}^2\mathrm{r}_0}}{\mathrm{c}^2\left(1-\frac{\mathrm{R_s}}{\mathrm{r}}-\frac{4\mathrm{G}\pi^2\rho_0\mathrm{r}^3}{3\mathrm{r}_0\mathrm{c}^2}\right)^2}\Pi_\mathrm{t}^2+\left(\frac{\mathrm{R_s}}{\mathrm{r}^2}-\frac{4\mathrm{G}\pi^2\mathrm{r}^2}{\mathrm{c}^2\mathrm{r}_0}\right)\Pi_\mathrm{r}^2-\frac{\Pi_\phi^2}{\mathrm{r}^3}\right], \tag{18}$$

$$\dot{\mathrm{t}} = \frac{\partial\mathcal{H}}{\partial\dot{\Pi}_\mathrm{t}} = \frac{1}{\mu\mathrm{c}^2}\left(1-\frac{\mathrm{R_s}}{\mathrm{r}}-\frac{8\mathrm{G}\pi^2\rho_0\mathrm{r}^3}{3\mathrm{r}_0\mathrm{c}^2}\right)^{-1}\Pi_\mathrm{t}, \tag{19}$$

$$\dot{\Pi}_\mathrm{t} = -\frac{\partial\mathcal{H}}{\partial\dot{\mathrm{t}}} = 0, \tag{20}$$

$$\dot{\phi} = \frac{\partial\mathcal{H}}{\partial\dot{\Pi}_\phi} = -\frac{\Pi_\phi}{\mu\mathrm{r}^2} \tag{21}$$

$$\dot{\Pi}_\phi = -\frac{\partial\mathcal{H}}{\partial\dot{\phi}} = 0, \tag{22}$$

From (20) and (22) we obtain

$$\Pi_\mathrm{t} = \alpha, \qquad \Pi_\phi = \ell, \tag{23}$$

where α and ℓ are the constants of motion.

Thus if the total energy is E, then from equation (16), it follows that,

$$2\mu\mathrm{E} = \left[\frac{\alpha^2}{\mathrm{c}^2}\left(1-\frac{\mathrm{R_s}}{\mathrm{r}}-\mathrm{kr}^3\right)^{-1}-\left(1-\frac{\mathrm{R_s}}{\mathrm{r}}-\mathrm{kr}^3\right)\Pi_\mathrm{r}^2-\frac{\mathrm{l}^2}{\mathrm{r}^2}\right] \tag{24}$$

where $k=\frac{8G\pi^2\rho_0}{3r_0c^2}$.

and the term

$$2\mu\mathrm{E} = \begin{Bmatrix} \mu^2\mathrm{c}^2 & \text{for particles} \\ 0 & \text{for photons} \end{Bmatrix} \tag{25}$$

Substituting the value of Π_r from equation (12) in (24), we have

$$\mu^2c^2 = \left[\frac{\alpha^2}{c^2}\left(1-\frac{R_s}{r}-kr^3\right)^{-1}-\mu^2\dot{r}^2\left(1-\frac{R_s}{r}-kr^3\right)^{-1}-\frac{l^2}{r^2}\right] \tag{26}$$

Rearranging gives,

$$\dot{\mathrm{r}}^2 = \frac{1}{\mu^2}\left[\frac{\alpha^2}{\mathrm{c}^2}-\frac{\mathrm{l}^2}{\mathrm{r}^2}\left(1-\frac{\mathrm{R_s}}{\mathrm{r}}-\mathrm{kr}^3\right)-\mu^2\mathrm{c}^2\left(1-\frac{\mathrm{R_s}}{\mathrm{r}}-\mathrm{kr}^3\right)\right] \tag{27}$$

to keep things simple, we make the change of variable from r to $\frac{1}{v}$

$$v = \frac{1}{r}, \qquad \rightarrow \qquad \dot{r} = \frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{v}\right) = -\frac{1}{v^2}\frac{du}{d\phi}\,\dot{\phi}, \tag{28}$$

With the change in variable, $k = \frac{8G\pi^2\rho_0}{3r_0c^2} = \frac{8G\pi^2\rho_0 v_0}{3c^2}$.

$$\left(\frac{dv}{d\phi}\right)^2 = \frac{u^4}{\mu^2\dot{\phi}^2}\left[\frac{\alpha^2}{c^2} - l^2v^2\left(1 - R_s v - \frac{k}{v^3}\right) - \mu^2c^2\left(1 - R_s v - \frac{k}{v^3}\right)\right] \tag{29}$$

Again substituting the value of $\dot{\phi}$ from equation (21) in (29), we get,

$$\left(\frac{dv}{d\phi}\right)^2 = \frac{1}{l^2}\left[\frac{\alpha^2}{c^2} - l^2v^2\left(1 - R_s v - \frac{k}{v^3}\right) - \mu^2c^2\left(1 - R_s v - \frac{k}{v^3}\right)\right] \tag{30}$$

taking the differential gives

$$\frac{d^2v}{d\phi^2} + v = \frac{\mu^2 m_s G}{l^2} + \frac{3m_s G}{c^2}v^2 - \frac{k}{2v^2} - \frac{6\mu^2c^2}{l^2v^4}k \tag{31}$$

we perform the change of variable

$$v = \frac{\mu^2 m_s G}{l^2}u = mu \tag{32}$$

where $m = \frac{\mu^2 m_s G}{l^2}$

Thus k changes as $k = \frac{8G\pi^2\rho_0}{3r_0c^2} = \frac{8G\pi^2\rho_0}{3c^2}mu_0 = k_1 m$.

where $k_1 = \frac{8G\pi^2\rho_0}{3c^2}u_0$

equation (31) becomes

$$m\frac{d^2u}{d\phi^2} + mu - m - \frac{3m_sG}{c^2}m^2u^2 + \frac{k_1 m}{2m^2u^2} + \frac{6\mu^2c^2k_1m}{l^2m^4u^4} = 0 \tag{33}$$

$$\frac{d^2u}{d\phi^2} + u - 1 - \epsilon u^2 + \frac{\delta}{u^4} + \frac{\lambda}{u^2} = 0 \tag{34}$$

Where ϵ, δ and λ are given by equation (35)

$$\left\{\begin{array}{l} \epsilon = \frac{3\mu^2 {m_s}^2 G^2}{l^2c^2} \\ \delta = \frac{6\mu^2c^2}{l^2}\frac{k_1}{m^4} = \frac{16\pi^2 l^6 u_0\rho_0}{\mu^6 m_s^4 G^3} \\ \lambda = \frac{k_1}{2m^2} = \frac{4\pi^2\rho_0 u_0 l^4}{3\mu^4 c^2 {m_s}^2 G} \end{array}\right\} \tag{35}$$

3.2 Method of Multiple Scales

Equation (34) is our final equation. We solve it by employing the technique of multiple scales. Let,

$$u = \sum_{i,j,k=0}^{\infty} u_{ijk}\epsilon^i \delta^j \lambda^k \tag{36}$$

and

$$\Phi_{i,j,k} = \epsilon^i \delta^j \lambda^k \phi \tag{37}$$

So by the chain rule,

$$\frac{d}{d\phi} = \sum_{i,j,k=0}^{\infty} \frac{d\Phi_{i,j,k}}{d\phi}\frac{\partial}{\partial \Phi_{i,j,k}} = \sum_{i,j,k=0}^{\infty} \epsilon^i \delta^j \lambda^k \frac{\partial}{\partial \Phi_{i,j,k}} \tag{38}$$

Then equation (34) becomes,

$$\begin{aligned}&\left(\frac{\partial}{\partial\Phi_{000}} + \epsilon\frac{\partial}{\partial\Phi_{100}} + \delta\frac{\partial}{\partial\Phi_{010}} + \lambda\frac{\partial}{\partial\Phi_{001}} + ...\right)^2 (u_{000} + \epsilon u_{100} + \delta u_{010} + \lambda u_{001}...)\\ &\quad + (u_{000} + \epsilon u_{100} + \delta u_{010} + \lambda u_{001}...) - 1 - \epsilon(u_{000} + \epsilon u_{100} + \delta u_{010} + \lambda u_{001}...)^2\\ &\quad + \delta(u_{000} + \epsilon u_{100} + \delta u_{010} + \lambda u_{001}...)^{-4} + \lambda(u_{000} + \epsilon u_{100} + \delta u_{010} + \lambda u_{001}...)^{-2} = 0\end{aligned} \tag{39}$$

For the first part we find the equation of order $\epsilon^0\delta^0\lambda^0$ which is given as,

$$\frac{\partial^2 u_{000}}{\partial {\Phi_{000}}^2} + u_{000} - 1 = 0 \tag{40}$$

The solution of equation (40) is

$$u_{000} = 1 + Be^{i\Phi_{000}} + \bar{B}e^{-i\Phi_{000}} \tag{41}$$

where B is a complex-valued function independent of Φ_{000}

The equation of order $\epsilon^1\delta^0\lambda^0$ is

$$\frac{\partial^2 u_{100}}{\partial {\Phi_{000}}^2} + u_{100} = -\frac{\partial^2 u_{000}}{\partial\Phi_{100}\partial\Phi_{000}} - \frac{\partial^2 u_{000}}{\partial\Phi_{000}\partial\Phi_{100}} + {u_{000}}^2 \tag{42}$$

To solve this equation, we choose the value of B in (41) such that the resonant term cancels out (we are only concerned about the coefficients of $e^{i\Phi 000}$ and $e^{-i\Phi 000}$). For that, we must have,

$$-2i\frac{\partial B}{\partial \Phi_{100}} + 2B = 0 \tag{43}$$

And its complex conjugate is

$$2i\frac{\partial \bar{B}}{\partial \Phi_{100}} + 2\bar{B} = 0 \tag{44}$$

Let $B = be^{i\beta}$, then solving from (43) and (44) gives b and β such that

$$\frac{\partial b}{\partial \Phi_{100}} = 0 \implies b \neq f(\Phi_{100}) \tag{45}$$

and $\beta = -\Phi_{100} + \gamma$ where γ is some function independent of both Φ_{000} and Φ_{100}.

The equation of order $\epsilon^0\delta^1\lambda^0$ is:

$$\frac{d^2 u_{010}}{d{\Phi_{000}}^2} + u_{010} = -\frac{\partial^2 u_{000}}{\partial \Phi_{000}\partial \Phi_{010}} - \frac{\partial^2 u_{000}}{\partial \Phi_{010}\partial \Phi_{00}} - \frac{1}{{u_{000}}^4} \tag{46}$$

To know the conditions such that the resonant terms in cancel out, we find the expression for $\frac{1}{{u_{00}}^4}$ first. For this, we explore the orthogonality of the complex exponential and use contour integration:

$$\frac{1}{2\pi}\int_0^{2\pi} e^{-i\Phi_{000}}(1 + Be^{i\Phi_{000}} + \bar{B}e^{-i\Phi_{000}})^{-4} d\Phi_{000} \tag{47}$$

putting $e^{i\phi_{00}} = z$ gives,

$$\frac{1}{2\pi i}\int \frac{dz}{z^2(1 + Bz + \bar{B}\frac{1}{z})^4} \tag{48}$$

$$\frac{1}{2\pi i B^4}\int \frac{z^2 dz}{(z - \alpha_{010})^4(z - \beta_{010})^4} \tag{49}$$

where α_{010} and β_{010} are the two roots. The pole exists at α_{010} only because $|\alpha_{010}| < 1$ and $|\beta_{010}| > 1$. The pole at α_{010} is of order 4.

Thus equation (47) leads to

$$= \lim_{z \longrightarrow \alpha_{010}} \frac{1}{3!B^4}(\frac{d^3}{dz^3})\frac{z^2}{(z - \beta_{010})^4} \tag{50}$$

Which gives,

$$= \frac{-4(1 + B\bar{B})B}{(1 - 4B\bar{B})^{\frac{7}{2}}} \tag{51}$$

Now the condition to cancel the resonant term in (46) is:

$$-2i\frac{\partial B}{\partial \Phi_{010}} + \frac{4(1 + B\bar{B})B}{(1 - 4B\bar{B})^{\frac{7}{2}}} = 0 \tag{52}$$

Notice that we aren't concerned with the expression in (46). We are only concerned with the coefficient of $e^{i\Phi_{000}}$. The coefficient of $e^{-i\Phi_{000}}$ is found by taking the complex conjugate.

$$2i\frac{\partial \bar{B}}{\partial \Phi_{010}} + \frac{4(1 + B\bar{B})\bar{B}}{(1 - 4B\bar{B})^{\frac{7}{2}}} = 0 \tag{53}$$

With these conditions (62) and (53), we put $B = be^{-i\Phi_{100}+i\gamma}$

From above equation, we get

$$\frac{d\gamma}{d\Phi_{010}} = -\frac{2(1+b^2)}{(1-4b^2)^{\frac{7}{2}}} \tag{54}$$

Solving for γ gives,

$$\gamma = -\frac{2(1+b^2)}{(1-4b^2)^{\frac{7}{2}}}\Phi_{010} + \eta \tag{55}$$

where η is a function independent of Φ_{000}, Φ_{100} and Φ_{010}

Similarly, the equation of order $\epsilon^0\delta^0\lambda^1$ is:

$$\frac{d^2u_{001}}{d\Phi_{000}{}^2} + u_{001} = -\frac{\partial^2 u_{000}}{\partial\Phi_{000}\partial\Phi_{001}} - \frac{\partial^2 u_{000}}{\partial\Phi_{001}\partial\Phi_{00}} - \frac{1}{u_{000}{}^2} \tag{56}$$

To know the conditions such that the resonant terms in (56) cancel out, we find the expression for $\frac{1}{u_{000}{}^2}$ first. We explore the orthogonality of the complex exponential and use contour integration like before:

$$\frac{1}{2\pi}\int_0^{2\pi} e^{-i\Phi_{000}}(1 + Be^{i\Phi_{000}} + \bar{B}e^{-i\Phi_{000}})^{-4} d\Phi_{000} \tag{57}$$

putting $e^{i\phi_{000}} = z$ gives,

$$\frac{1}{2\pi i}\int \frac{dz}{z^2(1 + Bz + \bar{B}\frac{1}{z})^2} \tag{58}$$

$$\frac{1}{2\pi i B^2}\int \frac{dz}{(z-\alpha_{001})^2(z-\beta_{001})^2} \tag{59}$$

where α_{001} and β_{001} are the two roots. The pole exists at α_{001} only because $|\alpha_{001}| < 1$ and $|\beta_{001}| > 1$. The pole at α_{001} is of order 2.

Thus equation (59) leads to

$$= \lim_{z\longrightarrow\alpha_{001}} \frac{1}{B^2}\frac{d}{dz}\left(\frac{1}{(z-\beta_{001})^2}\right) \tag{60}$$

Which gives,

$$= \frac{-2B}{(1-4B\bar{B})^{\frac{3}{2}}} \tag{61}$$

Now the condition to cancel the resonant term in (56) is given by:

$$-2i\frac{\partial B}{\partial\Phi_{001}} + \frac{2B}{(1-4B\bar{B})^{\frac{3}{2}}} = 0 \tag{62}$$

Notice that we aren't concerned with the expression in (56). We are only concerned with the coefficient of $e^{i\Phi_{000}}$. The coefficient of $e^{-i\Phi_{000}}$ is found by taking the complex conjugate.

$$2i\frac{\partial \bar{B}}{\partial \Phi_{001}} + \frac{2\bar{B}}{(1-4B\bar{B})^{\frac{3}{2}}} = 0 \tag{63}$$

With these conditions (62) and (63), we can confidently put $B = be^{-i\beta}$

where

$$\beta = -\Phi_{100} + \gamma \tag{64}$$

$$= -\Phi_{100} + \frac{2(1+b^2)}{(1-4b^2)^{\frac{7}{2}}}\Phi_{010} + \eta \tag{65}$$

So we put:

$$B = b.e^{-i(-\Phi_{100} + \frac{2(1+b^2)}{(1-4b^2)^{\frac{7}{2}}}\Phi_{010} + \eta)} \tag{66}$$

where η is a function independent of Φ_{000}, Φ_{100} and Φ_{010} Solving this using equation (62), we get

$$\eta = (1-4B\bar{B})^{\frac{-3}{2}}\Phi_{001} + \kappa \tag{67}$$

where κ is independent of Φ_{000}, Φ_{100}, Φ_{010} and Φ_{001}

Putting $b = \frac{e}{2}$ in both equations (55) and (67), we write the final expression for u as:

$$u = 1 + ecos\left[\left(1 - \epsilon - 2\delta(1+\frac{e^2}{4})(1-e^2)^{-\frac{7}{2}} + \lambda(1-e^2)^{\frac{-3}{2}}\right)(\phi - \phi_0)\right] \tag{68}$$

Equation (68) is the correction due to the surface tension on the precession of the perihelion of mercury.

The angle of precession θ over the course of one orbit is

$$\theta = 2\pi\left(\epsilon - 2\delta(1+\frac{e^2}{4})(1-e^2)^{-\frac{7}{2}} + \lambda(1-e^2)^{\frac{-3}{2}}\right) \tag{69}$$

CHAPTER 4

RESULTS & DISCUSSION

Chapter 4

Results and Discussion

Now we check if the correction given by ϵ in equation (68) and (73) agrees with the numerical value as predicted by General theory of relativity. If the δ and λ correction can account for the discrepancy between theory and observation, our model is successful.

4.1 Correction for GR

Taking into account that General Relativity predicts for the perihelion shift with $l^2 = \mu^2 m_s Ga(1-e^2)$, and, using the data for Mercury planet, we have:

$$\begin{aligned} \mathrm{G} &= 6.67259\times10^{-11}\frac{\mathrm{m}^3}{\mathrm{Kg\,s}^2}, \qquad \mathrm{c}^2 = 8.98755179\times10^{16}\frac{\mathrm{m}^2}{\mathrm{s}^2}, \qquad \mathrm{m_s} = 1.9891\times10^{30}\mathrm{Kg}, \\ \mathrm{a_m} &= 57.909175\times10^{9}\mathrm{m}, \qquad \mu_\mathrm{m} = 3.3022\times10^{23}\mathrm{Kg}, \qquad \mathrm{e_m} = 0.205630, \end{aligned} \tag{70}$$

$$\Delta\phi_{GR} = 2\pi\epsilon = 2\pi\frac{3\mu^2 {m_s}^2 G^2}{l^2c^2} = 2\pi\frac{3m_sG}{c^2a(1-e^2)}, \quad \to \quad \Delta\phi_{GR} = 2\pi(7.98815722\times10^{-8})\frac{rad}{rev} \tag{71}$$

per 414.92164 orbits per century, with 1" $= \frac{2\pi}{1296000}$rad

$$\Delta\phi_{\mathrm{GR}} = \frac{42.95539245"}{\mathrm{century}} \tag{72}$$

taking in account that the perihelion shift observed data is $\Delta\phi_{\mathrm{obs}} = 43.1\pm0.5$, then

$$\delta\phi = \Delta\phi_{\mathrm{obs}} - \Delta\phi_{\mathrm{GR}} = \frac{0.144608"}{\mathrm{century}}$$

.

4.2 Correction for Surface Tension

$$\Delta\phi_{ST} = 2\pi\left|\left(-2\delta(1+\frac{e^2}{4})(1-e^2)^{-\frac{7}{2}} + \lambda(1-e^2)^{\frac{-3}{2}}\right)\right| \tag{73}$$

Substituting values of δ and λ given by

$$\delta = \frac{16\pi^2a^3(1-e^2)^3\rho_0u_0}{m_s} \tag{74}$$

and

$$\lambda = \frac{4\pi^2\rho_0 u_0 G a^2 (1-e^2)^2}{3c^2} \tag{75}$$

$$\Delta\phi_{ST} = 2\pi \left| \left(-2\frac{16\pi^2 a^3 \rho_0 u_0}{m_s(1-e^2)^{-\frac{1}{2}}}(1+\frac{e^2}{4}) + \frac{4\pi^2\rho_0 u_0 G a^2 (1-e^2)^{\frac{1}{2}}}{3c^2} \right) \right| \tag{76}$$

Putting $\Delta\phi_{ST}$ in equation 76), we obtain the value of Density as, $\rho_0 = 3.43 \times 10^{-15} kg/m^3$

For comparison,

Density of outer corona is $\rho = 10^{-15} kg/m^3$

Amount (arcsec/Julian century)	Cause
532.3035	Gravitational tugs of other solar bodies
0.0286	Oblateness of the Sun (Quadrapule moment)
42.9799	Gravitoelectric effects (Schwarzschild-like), a General Relativity effect
−0.0020	Lense-Thirring Precession
575.31	Total predicted
574.10±0.65	Observed
43.1±0.5	General Relativity effect including surface forces

FIG. 4: Different Sources of the Precession of the Perihelion of Mercury

CHAPTER 5

SCOPE & FUTURE IMPROVEMENTS

Chapter 5

Scope and Future Improvements

5.1 Qualitative aspects of the work

We have extended the formalism of GR to include surface forces. And not only that, this framework explains many observational effects like precession of perihelion of mercury and limb shift. Further, this work can be extended to make an explanation of the redshift. If it is possible for photons to lose energy during their travel, an alternative cosmology can be re-written entirely. [6]

References

[1] F. W. Dyson, A. S. Eddington, and C. Davidson, Philosophical Transactions of the Royal Society of London Series A **220**, 291 (1920).

[2] J. Halm, Astronomische Nachrichten **173**, 273 (1907).

[3] W. S. Adams, APJ **31**, 30 (1910).

[4] M. Murga, G. Zhu, B. Ménard, and T.-W. Lan, **452**, 511 (2015), 1503.02697.

[5] M. G. Adam, mnras **108**, 446 (1948).

[6] F. Zwicky, Proceedings of the National Academy of Science **15**, 773 (1929).

[7] P. T. Bruggencate, Proceedings of the National Academy of Sciences of the United States of America **16**, 111 (1930), ISSN 00278424, URL `http://www.jstor.org/stable/85473`.

[8] C. Ortiz, Int. J. Mod. Phys. D **29**, 2050115 (2020), 2011.02317.